FACULTAD DE CIENCIAS EMPRESARIALES Y ECONOMIA

Serie de documentos de trabajo del Departamento de Economía /
Department of Economics Working Papers Series

FAIRNESS AND REDISTRIBUTION, A COMMENT

Rafael Di Tella
Harvard Buisness School
rditella@hbs.edu

Juan Dubra
Departamento de Economía
Facultad de Ciencias Empresariales y Economía
Universidad de Montevideo
jdubra@um.edu.uy

Working paper UM_CEE_2013-02
http://www.um.edu.uy/cee/investigaciones/

Fairness and Redistribution, a Comment

By RAFAEL DI TELLA AND JUAN DUBRA*

We provide an example that shows that in the Alesina and Angeletos (2005) model one can obtain multiplicity even if luck plays no role in the economy. Thus, it is not critical that the noise to signal ratio be increasing in taxes, or that desired taxes are increasing in the noise to signal ratio.
JEL: D31, E62, H2, P16.
Keywords: Inequality, taxation, redistribution, political economy.

In an influential paper Alesina and Angeletos (2005), henceforth AA, argued that a preference for fairness could lead two identical societies to choose different economic systems. In particular, two equilibria might arise: one with low taxes and a belief that the income-generating process is "fair" because effort is important (an "American" equilibrium) and another with high taxes and the belief that the process is "unfair" because luck prevails. Piketty (1995) had shown that a similar pattern could arise from standard preferences if initial beliefs about the relative importance of effort and luck in generating income differed across the two societies, while Benabou and Tirole (2006) study this issue using more realistic preferences (Buera et al. (2011) discusses the evolution of beliefs about economic systems). A key contribution of AA is to obtain these two equilibria from identical societies assuming agents prefer outcomes that are fair, an important modification because fairness considerations seem central in the demand for redistribution and because in several settings (as in some ultimatum games) such preferences for fairness can lead to large (material) inefficiencies.

In this note we report a difficulty we encountered when interpreting the results in AA: we find multiplicity (and demand for redistribution) even if luck plays no role. In other words, there is multiplicity even if the equilibrium tax rate is independent of the signal to noise ratio (a quantity that expresses how important is effort, relative to luck, in the determination of income). This conflicts with the notion that the signal to noise ratio plays a central role in generating multiplicity with AA preferences for fairness.

I. The AA model.

The economy is populated by a measure 1 continuum of individuals $i \in [0, 1]$, who live for two periods: in the first period the individuals accumulate capital; in the middle of their lives the taxes are set; in the second period, individuals exert effort (work). Total

* Di Tella: Harvard Business School, 25 Harvard Way Boston, MA 02163, United States, rditella@hbs.edu. Dubra: Departamento de Economía, Universidad de Montevideo, Prudencio de Pena 2440, CP 11.600, Montevideo, Uruguay, dubraj@um.edu.uy. We thank Julio Rotemberg for his comments. We thank the Canadian Institute for Advanced Research, the Fondo Clemente Estable, and ANII.

pre-tax lifetime income is

$$y_i = A_i\left[\alpha k_i + (1-\alpha)\, e_i\right] + \eta_i \tag{1}$$

where A is talent, k is the capital accumulated during the first period, e is effort during the second period, η is "noise" or "luck", and $\alpha \in (0,1)$ is a technological constant. The government imposes a flat tax rate τ on income and redistributes the proceeds in a lump sum fashion, so that the individual's consumption is, for government transfer $G = \tau \int_i y_i$,

$$c_i = (1-\tau)\, y_i + G.$$

Individual preferences are, for $u_i = V_i\left(c_i, k_i, e_i\right) = c_i - \frac{1}{2\beta_i}\left[\alpha k_i^2 + (1-\alpha)\, e_i^2\right]$,

$$U_i \equiv u_i - \gamma\,\Omega \equiv c_i - \frac{1}{2\beta_i}\left[\alpha k_i^2 + (1-\alpha)\, e_i^2\right] - \gamma\,\Omega$$

where u_i is private utility from own consumption, investment and effort, β_i is an impatience parameter, γ is "distaste for unfair outcomes" and Ω is a measure of the social injustice in the economy. AA assume that A, η and β are iid across agents, and that for $\delta = A^2\beta$, $Cov\,(\delta, \eta) = 0$. We let $\overline{\delta}$ be the mean of δ, and δ_m its median; AA also assume $\Delta = \overline{\delta} - \delta_m \geq 0$ and normalize $\delta_m = 2$. Similarly, $\overline{\eta}$ is the mean of η and η_m its median.

AA define social injustice as $\Omega = \int_i \left(u_i - \widehat{u}_i\right)^2$, where u_i is the actual level of private utility, and $\widehat{u}_i$ is a measure of the "fair" level of utility the individual should have (deserves) on the basis of his talent and effort. They define $\widehat{u}_i = V_i\left(\widehat{c}_i, k_i, e_i\right)$ for

$$\widehat{c}_i = \widehat{y}_i = A_i\left[\alpha k_i + (1-\alpha)\, e_i\right]. \tag{2}$$

The individual chooses k when taxes haven't been set, so anticipating a tax rate of τ_e (which will be equal to the actual τ in equilibrium) he maximizes

$$u_i = (1-\tau_e)\, A_i\left[\alpha k_i + (1-\alpha)\, e_i\right] + (1-\tau_e)\,\eta_i + G - \frac{1}{2\beta_i}\left[\alpha k_i^2 + (1-\alpha)\, e_i^2\right] \tag{3}$$

with respect to k, and using the actual tax rate in equation (3) maximizes with respect to e to obtain

$$k_i = (1-\tau_e)\, A_i\beta_i \quad \text{and} \quad e_i = (1-\tau)\, A_i\beta_i. \tag{4}$$

Then, $U_i = u_i - \gamma\,\Omega$ implies

$$\begin{aligned} U_i\left(\tau, \tau_e\right) \;=\; & \frac{\delta_i}{2}\left(1 - \alpha\tau_e^2 - (1-\alpha)\,\tau^2\right) + \eta_i + \tau\left(\overline{\eta} - \eta_i\right) + \tau\left(\overline{\delta} - \delta_i\right)\left[1 - \alpha\tau_e - \tau\,(1-\alpha)\right] \\ & -\gamma\,\left((1-\tau)^2\,\sigma_\eta^2 + \tau^2\left[1 - \alpha\tau_e - (1-\alpha)\,\tau\right]^2\sigma_\delta^2\right) \end{aligned}$$

A. Example: multiplicity without luck

AA say:

"The critical features of the model that generate equilibrium multiplicity are (a) that the optimal tax rate is decreasing in the signal-to-noise ratio and (b) that the equilibrium signal-to-noise ratio is in turn decreasing in the tax rate."

We now present an example with no noise, no luck, and therefore a constant noise-to-signal ratio, that still has multiple equilibria.

Set $\delta_m = 2$, $\overline{\delta} = \frac{47}{20}$, $\overline{\eta} = \eta_m = 0$, $\gamma\sigma_\delta^2 = \frac{27}{25}$, $\alpha = \frac{999}{1000}$ and $\sigma_\eta^2 = 0$. We first note that $\frac{dU_m(\tau_e,\tau_e)}{d\tau} = 0$ has three solutions for $\tau_e^h \approx 0.99308$, $\tau_e^m \approx 0.8029$ and $\tau_e^l \approx 0.2031$. The existence of three roots in $[0, 1]$ follows from Bolzano's Theorem and $\frac{dU_m\left(\frac{1}{5},\frac{1}{5}\right)}{d\tau}, \frac{dU_m\left(\frac{9}{10},\frac{9}{10}\right)}{d\tau} > 0 > \frac{dU_m\left(\frac{1}{2},\frac{1}{2}\right)}{d\tau}, \frac{dU_m(1,1)}{d\tau}$.

This means, in principle, but we will now check it, that given an expected tax rate of τ_e^j for $j = l, m, h$, the tax rate that maximizes the utility of the voter with the median values of the shocks is τ_e^j; that is, there is multiplicity of equilibria, even though luck plays no role.

We now check that given a tax rate of τ_e^l the tax rate that maximizes the utility of the individual with the median values of the shocks is again $\tau = \tau_e^l$ (the cases of τ_e^m and τ_e^h are similar and omitted). First note that the optimal tax rates for τ_e^l are neither 0 nor 1, since $U_m\left(1, \frac{20302}{100000}\right) < U_m\left(0, \frac{20302}{100000}\right) < U_m\left(\frac{20302}{100000}, \frac{20302}{100000}\right)$, and continuity of $U_m(\tau, \tau_e)$ in τ_e implies that for τ_e^l close to $\frac{20302}{100000}$ we still have $U_m\left(1, \tau_e^l\right) < U_m\left(0, \tau_e^l\right) < U_m\left(\tau_e^l, \tau_e^l\right)$. Therefore, the tax $\tau \in [0, 1]$ that maximizes $U_m\left(\tau_e, \tau_e^l\right)$ must solve $dU\left(\tau_e, \tau_e^l\right)/d\tau = 0$. We know that $dU\left(\tau_e^l, \tau_e^l\right)/d\tau = 0$ (by definition of τ_e^l), so we only need to check that it is the global maximum among $\tau \in [0, 1]$ which is ensured by concavity in the domain: $d^2U_m\left(\tau, \tau_e^l\right)/d\tau^2 \approx -1.296 \times 10^{-5}\tau^2 + 1.0331 \times 10^{-2}\tau - 1.3781 < 0$ (for all $\tau \in [0, 1]$).

B. Discussion

Note that with no luck in the model, $\sigma_\eta^2 = 0$, for $\tau_e = \tau$ we obtain that for $\sigma_{\hat{y}}^2$ the variance of "fair" income (the signal in AA), $\Omega = \tau^2\sigma_{\hat{y}}^2 = \sigma_\delta^2\tau^2(1-\tau)^2$ which is non-monotonic in τ, while one might expect unfairness to increase with taxes.[1] Hence, it is possible that the key insights in AA can be restored if other definitions of what is fair are used. For example, one alternative definition involves keeping taxes in the definition of fair consumption (in AA "fair" consumption involves no taxes and no luck).[2] Alesina and Cozzi (2012) analyze multiplicity using another approach where the AA

[1]Thus, a tax rate of $\tau = 1$ also minimizes unfairness Ω, which seems counterintuitive since there is no luck in this economy. Moreover, one difficulty in evaluating the claim that the tax rate that minimizes Ω depends on the signal to noise ratio, is that the signal also appears to depend on the tax.

[2]Di Tella, Dubra and MacCulloch (2010) take this approach (see also the comment by Angeletos, 2010). Alesina et al. (2010) study the dynamic implications of both types of preferences and note how the definition of fairness in AA is not only about fairness, but reflects insteat that individuals "tolerate inequality coming from innate ability and effort, but are averse to inequality arising from everything else, luck and redistribution."

preferences are normalized by average income. Another possibility would be to insist that the effort imputed in "fair" consumption takes into account that there are no taxes. In other words, it may be more reasonable to modify AA so that the k_i and e_i used to substitute into $\widehat{y}_i$ in equation (2), are not those associated to the case where taxes may be positive. Finally, one may also insist on preferences for fairness that are consistent with the empirical evidence. For example, Levine (1998) and Rotemberg (2008) demonstrate that preferences for "reciprocal altruism" are consistent with the available evidence from the ultimatum games, while Di Tella and Dubra (2012) show that they lead to multiplicity in an economy similar to that presented in AA.

One difficulty for exploring these conjectures in the AA framework is that a counter example to the main theorem can be produced because AA claim that the individual with the median values of the shocks is the median voter, but in general he is not. In the online appendix we give an example where the equilibrium tax rate, the one preferred by the median voter, is not the one identified in AA. The tax rate identified as the equilibrium in AA would be defeated in voting by the one preferred by the median voter (which can be shown to be a Condorcet winner, even if the Median Voter Theorem does not apply). This wedge between the prediction of the AA model and what would happen in that economy is relevant, since it is currently not known if in the AA model multiplicity can arise when the equilibrium tax rate is one that, when anticipated, maximizes the utility of the median voter (in one special case, when $\overline{\delta} = \delta_m$, Di Tella *et al.* show how to analyze the AA model, establishing that the median voter's preferred tax rate is a Condorcet winner. But this case is not very relevant empirically, since it implies mean income equal to median income, and does not allow for a Meltzer-Richard effect[3]).

In brief, we believe that the main point in AA, namely that a preference for fairness can lead to multiple equilibria, is potentially valid but some aspects of the particular framework they propose need to be revised.

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[3]If there was multiplicity in this case, a continuity argument would establish the existence of multiple equilibria even if $\overline{\delta} > \delta_m$. But two questions would remain: the argument would not apply to any $\overline{\delta} > \delta_m$, and $\overline{\delta}$ close enough to δ_m might not be enough to fit real data; once the assumption of $\overline{\delta} = \delta_m$ is dropped, we do not know whether the median voter's preferred tax rate is still a Condorcet winner, and so the plausibility of such a tax rate arising as an equilibrium is reduced.

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Online Appendix

AA define the government’s choice of policy in p. 967: “The optimal policy maximizes the utility of the median voter.” They then identify the median voter as the individual with the median values of the shocks: “Assuming that luck has zero mean and median, the median voter, denoted by $i = m$, is an agent with characteristics $\delta_m = median\,(\delta_i)$ and $\eta_m = 0$.” In general, however, the median voter is not the individual with the median values of the shocks. Due to this wedge, one can produce a counterexample to Theorem 1.

AA define an equilibrium as a tax policy τ^* such that when the anticipated policy is $\tau_e = \tau^*$, the ex-post optimal policy (the one preferred by the median voter) is τ^*. Theorem 1 then says that an equilibrium always exists and corresponds to any fixed point of

$$f(\tau_e) = \arg\min_{\tau\in[0,1]} \left\{ \begin{array}{c} \frac{\delta_m}{2}(1-\alpha)\tau^2 + \tau^2(1-\alpha\tau_e-(1-\alpha)\tau)^2\gamma\sigma_\delta^2 + \\ (1-\tau)^2\gamma\sigma_\eta^2 - \tau(1-\alpha\tau_e-(1-\alpha)\tau)\left(\overline{\delta}-\delta_m\right) \end{array} \right\}$$

where $f(\tau_e)$ is the set of tax rates that maximize the utility of the individual with the median values of the shocks.

We now show that for some distributions and parameter values, 0 is an equilibrium, but it is not a fixed point of f : with an expected tax rate of $\tau_e = 0$, the median voter’s preferred tax rate is 0, and if the government maximizes his utility, then it chooses $\tau = 0$, so that it is an equilibrium. In contrast, when the expected tax rate is 0, the utility of the individual with the median values of the shocks is maximized for $f(0) = 1$, showing that 0 is not a fixed point of f.

First, set $\gamma = 0$, $\alpha = \frac{442}{443}$ and $\overline{\delta} = \frac{819}{400}$ and $\delta_m = 2$.[4] Note that if $\tau_e = 0$, we obtain that

[4]It is easy to build counterexamples to the theorem assuming $\gamma > 0$, but the calculations are simpler with $\gamma = 0$, and

f minimizes

$$(1-\alpha)\,\tau^2 - \tau\,(1-(1-\alpha)\,\tau)\,\frac{19}{400}$$

which implies $f(0)=1$. Hence, what we will show to be an equilibrium, $\tau_e=\tau=0$, is not a fixed point of f (this is not a consequence of having chosen $\gamma=0$).

We now give distributions of δ and η satisfying these restrictions, but for which the equilibrium tax rate is $\tau_e=\tau=0$, but 0 is not a fixed point of f.

The distributions δ_i and η_i are

$$p_\delta(x)=\begin{cases}\frac{19}{40} & x=\frac{19}{10}\\ \frac{2}{40} & x=2\\ \frac{19}{40} & x=\frac{22}{10}\end{cases} \qquad \text{and} \qquad p_\eta(x)=\begin{cases}\frac{1}{10} & x=\frac{61}{400}\\ \frac{8}{10} & x=0\\ \frac{1}{10} & x=-\frac{61}{400}\end{cases}.$$

We assume also that δ_i and η_i are independent. In this case, the mean and median of η are 0. Also, the median of δ is $\delta_m=2$ and its mean is $\frac{819}{400}$.

Since $\gamma=0$, and $2\overline{\delta}>\max\delta_i$, preferences are single peaked and the optimal tax rate τ^* for a person $\left(\delta_i,\eta_i\right)$ is given by $dU_i/d\tau=0$, or

(A1)

$$\frac{dU_i}{d\tau}=\left(\overline{\delta}-\delta_i\right)(1-\alpha\tau_e-\tau\,(1-\alpha))-\tau\overline{\delta}\,(1-\alpha)-\eta_i\Rightarrow\tau^*=\frac{\left(\overline{\delta}-\delta_i\right)(1-\alpha\tau_e)-\eta_i}{(1-\alpha)\left(2\overline{\delta}-\delta_i\right)}$$

or 0 if $\tau^*<0$ or 1 if $\tau^*>1$. From equation (A1), and the border conditions $1\geq\tau\geq0$ we get that the optimal tax rates for each combination of shocks, if $\tau_e=0$, is given by

$\eta \setminus \delta$	$\frac{19}{10}$	2	$\frac{22}{10}$	Pr
$\frac{61}{400}$	0	0	0	$\frac{1}{10}$
0	1	1	0	$\frac{8}{10}$
$-\frac{61}{400}$	1	1	0	$\frac{1}{10}$
Pr	$\frac{19}{40}$	$\frac{2}{40}$	$\frac{19}{40}$	

A tax rate of 0 accumulates the votes of $\frac{211}{400}\%$ of the population, and therefore "the" median voter is any voter whose preferred tax rate is 0 (and not the individual with shocks $(\delta,\eta)=(2,0)$, whose preferred tax rate is 1, as claimed in AA).

One possible solution is to assume that shocks in AA are symmetric, so as to ensure that the median voter is the individual with the median values of the shocks. It is possible then to show that, although preferences are not single peaked, this individual's preferred tax rate is a Condorcet winner (see Di Tella, *et al.* 2010, and Di Tella and Dubra 2011). In this case, however, the model can no longer capture a Meltzer-Richard motive for redistribution.

$\gamma=0$ is allowed by AA.