BUSINESS CYCLES AND THE CHOICE OF MONETARY POLICY TARGETS WITH CENTRAL BANK LEARNING

Victor Pacharoni*
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ABSTRACT

In the “new open economy” macro literature, the role of monetary policy focuses on Taylor rules, with some “stickiness” in price or wage dynamics. However this paper incorporates a “learning” process on the side of the Central Bank. This paper uses a stochastic dynamic general equilibrium model for small open economy, with terms of trade shocks and different degrees of information stickiness. This paper has produces results showing that the central bank should target another variable (besides inflation) to improve welfare. The result is on based on incomplete pass-through of exchange rate and stickiness in the Central Bank “learning” the true model. However this model has neither labor decisions nor productivity shocks. This model shows that if there are more sources of “noise” in the economy, a second variable in the policy function improves welfare.

RESUMEN

En la literatura de la “nueva macroeconomía abierta”, el rol de la política monetaria se enfoca en la regla de Taylor, con precios o salarios “pegajosos”. Sin embargo este artículo incorpora un proceso de “aprendizaje” por parte del Banco Central. El trabajo usa un modelo dinámico de equilibrio general estocástico para pequeñas economías abiertas, con movimientos en los Términos de Inter-cambio y diferentes tiempos para conocer la información. El resultado de este artículo muestra que el Banco Central debe tener por objetivo otra variable, además de la inflación, para mejorar el bienestar. El resultado está basado en el pasaje incompleto de tasa de cambio y el tiempo en el Banco Central “aprende” el verdadero modelo. Sin embargo este modelo no tiene en cuenta, ni decisiones de trabajo, ni perturbaciones de la productividad. Este modelo muestra que si hay más fuentes de ruido en la economía, una segunda variable en la función de política mejora el bienestar.

*Universidad Católica de Córdoba, Argentina. Email: victorp@uccor.edu.ar

INTRODUCTION

In the Real Business Cycle (RBC) approach to open economics, large and recurrent fluctuations in terms of trade are widely viewed as an important driving force of business cycles. The terms of trade affect the industrial countries mainly by raising the relative price of energy. In developing countries this effect primarily affects the price of imported capital goods.

In the literature of the “new open economy” macro, the role of monetary policy focuses on Taylor rules, with some “stickiness” in price or wage dynamics. However there is another form of stickiness, and that is in the form of “sticky information”. Mankiw and Reis (2002) contend that this form of stickiness displays properties that are more consistent with accepted views about monetary policy than the commonly used sticky price model. One form of sticky information is when one or more agent has to “learn” the process of the economy.

Many of the papers in this literature focus on the learning process in the private sector and how they need to learn the behavior of public sector, specially the central bank policy rule. (Bullard and Metra, 2002, and Orphanides and Williams, 2003). Following Sargent (1999) and Lim and McNelis (2003), this paper incorporates a “learning” process on the side of the central bank. This means that the central bank does not know the true “laws of motion” of the macroeconomic variables generated by the private sector.

Asset price booms and busts have been important factors in macroeconomic fluctuations in both industrial and developing countries. Developments in asset markets can have a significant impact on both inflation and real economic activity. Bernanke and Gertler (2001) argue that it is appropriate that the central bank respond to them. However they contend that changes in asset prices should affect monetary policy only to the extent that they affect the central bank’s forecast of inflation. Moreover Cecchetti, Genberg, Lipsky and Wadhwani (2002) argue that the central bank can achieve superior performance by adjusting its policy instruments not only in response to its forecast of future inflation and the output gap, but to asset prices as well.

Tobin’s $q$ is used to account for the price of replacement of the capital. Tobin’s $q$ is the ratio of the market value of an additional unit of capital to its replacement cost. Hayashi (1982) shows that the average and the marginal values of capital are identical under neoclassical assumptions. Tobin and Brainard (1977) conjecture that the average and marginal values respond in similar ways to shocks, and the empirical evidence in Abel and Blanchard (1986) appears to support the conjecture. The stock market indices are the value for the Tobin’s $q$ used in this research.

The specific aim of the paper is to examine the welfare implications of incorporating a second key variable output growth or asset price volatility to determine the interest rate policy in a small open economy subject to terms of trade shocks with the central bank facing limited information.

This paper examines different scenarios where the central bank applies different policy targets in a stochastic dynamic general equilibrium model subject to terms of trade shocks. The behavior of the private sector is consistent with forward-looking rational expectations.
The welfare implications are evaluated in different scenarios according to two different stickiness levels: first, in the learning process of the central bank, and secondly, in the degree of “pass-through” of the exchange rate.

The central bank needs to learn about the laws of motion of goods prices, output, and asset prices from past data, through least squares regression period by period. Using this information, it updates the policy rule by a linear quadratic optimization. The monetary authority is thus “boundedly rational”, in the sense of Sargent (1999), where “bounded” means misspecification of the model and “rational” describes the use of least squares. The other scenario is when the central bank follows an “optimal” Taylor rule for each target. In this case, the central bank uses a rule with optimal weights on past inflation, growth, and asset prices. The weights are selected to maximize welfare.

Following Lim and McNelis (2003), the “pass-through” of the price of imported goods is determined according to the local currency pricing formulation of Betts and Devereux (2000). This paper analyzes only two cases, full and low pas-through.

The methodology used here is a combination of parameterized expectation algorithm (used for the private sector) with a linear quadratic approach when the central bank updates its forecast in the learning process. Aruoba, Fernández-Villaverde and Rubio Ramírez (2003) show that polynomials are better than Finite Elements or perturbation methods when there are large shocks. Duffy and McNelis (2001), show that the neural network approach gives better approximation than the polynomials with the same number of parameters, or equivalent accuracy with less parameters.

The next Section describes the model, for the private sector and the monetary authority. Section 3 presents equilibrium and dynamic programming formulations. Section 4 describes the parameters and initial conditions. Section 5 presents the results of the simulations and Section 6 concludes.

1 MODEL

The framework of analysis contains two modules - a module which describes the behavior of the private sector, and module which describes the behavior of the central bank.

1.1 Private Sector Behavior

There are three parts in this subsection: The first is consumption, derived from the utility function, and the demand side of the economy. The second is production from which I derive investment. The third gives constraints of the representative household-firm, of the government and of the country.

1.1.1 Consumption

In this economy, there is a representative agent who lives forever. The preferences are given by maximizing the welfare, the present value of expected utility given by a composite good, $C_t$, and subjective endogenous discount factor, $\xi_t$. 

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The symbol $E_t$ is the expectations operator, conditional on information available at time $t$, while $\beta$ approximates the elasticity of the endogenous discount factor, $\xi_t$, with respect to the average consumption index, $\bar{C}$. The endogenous discount factor is due to Uzawa (1968) and Mendoza (1991, 1995). The main reason for using the endogenous discounting factor is that it produces well behaved dynamics with deterministic stationary equilibria in an open economy.

This paper uses a modified endogenous discount function following Schmitt-Grohé and Uribe (2001). The individual agent’s discount factor does not depend on the agent’s consumption, but rather the discount factor depends on the average level of past consumption. In equilibrium, the individual consumption index and the average index are identical, $C_t = \bar{C}_t$.

With that, the representative agent takes the endogenous discount factor, as given during the maximization problem.

The composite good has the following structure:

\[
C_t = C_{\tau,t}^{1-\theta} C_{n,t}^{\theta} \\
C_{\tau,t} = C_{x,t}^{1-\alpha} C_{m,t}^{\alpha}
\]

Tradeable, ($C_{\tau}$) and non-tradeable, ($C_{n}$), goods are expressed in Cobb-Douglas unitary-elasticity form where $\theta$ is the share in traded goods. Tradeable goods are also expressed in a Cobb-Douglas function between exportable, ($C_x$), and importable, ($C_m$), goods where $\alpha$ is the share in exportable goods. The aggregate consumption expenditure constraints are given by the following equations:

\[
P_t C_t = P_{\tau,t} C_{\tau,t} + P_{n,t} C_{n,t} \\
P_{\tau,t} C_{\tau,t} = P_{x,t} C_{x,t} + P_{m,t} C_{m,t}
\]

where $P_{\tau}, P_{n}, P_{x}, P_{m}$ are the prices in the economy, in traded, in non-traded, exportable and importable goods respectively. The definition of the real exchange rate ($Z_t$), and the terms of trade, ($J_t$), are:

\[
Z_t = \frac{P_{\tau,t}}{P_{n,t}} \\
J_t = \frac{P_{x,t}}{P_{m,t}}
\]
1.1.2 Production

There are two production sectors: export and non-traded. Production of exports, \( Y_x \), has a Cobb-Douglas technology:

\[
Y_{x,t} = A_{x,t} K_t^{1-\alpha_x}
\]

where \( A_{x,t} \) represents the labour factor productivity term in the production of export goods and \( K_t \) is the amount of capital. The production of non-traded goods, \( Y_n \), which is usually in services, is given by the labour productivity term, \( A_{n,t} \):

\[
Y_{n,t} = A_{n,t}
\]

Capital has a depreciation rate, \( \delta \), and evolves according the following identity:

\[
K_{t+1} = (1 - \delta) K_t + I_t
\]

where \( I_t \) represents “net” investment. The “gross” investment is the sum of the “net” investment plus a quadratic adjustment cost function for accumulation of capital, \( \frac{\chi}{2K_t} I_t^2 \).

The parameter \( \chi \) governs the adjustment cost of capital. This specific cost function shows an increasing marginal cost of investment. This assumption captures the observation that a faster pace of change requires a greater than proportional rise in installation costs. This function also has the advantage of equal marginal and average shadow prices for capital. (see Obstfeld and Rogoff, 1996).

1.1.3 Constraints

- Agent’s Budget Constraint

The representative household-firm has the following budget constraint:

\[
(1 + \tau) C_t + \frac{\tau}{\bar{P}_t} (L_t - L_{t-1} (1 + \bar{i}_{t-1})) + \frac{1}{\bar{P}_t} (B_t - B_{t-1} (1 + \bar{i}_{t-1}) + \bar{\tau}_{t-1}) = \frac{P_{x,t}}{\bar{P}_t} A_{x,t} K_t^{1-\alpha_x} + \frac{P_{n,t}}{\bar{P}_t} A_{n,t} - \frac{P_{m,t}}{\bar{P}_t} I_t - \frac{P_{m,t}}{\bar{P}_t} \frac{\chi}{2K_t} I_t^2
\]

where there are two different taxes: a fixed proportion of the consumption, \( \tau \), and a sum lump tax, \( \bar{\tau}_t \), period by period. The nominal exchange rate, \( \bar{S} \), translates the international prices to domestic ones. The household-firm may lend to the domestic government and accumulate bonds, \( B_t \), which pay the nominal interest rate, \( i_t \), given by the central bank. They can also borrow and accumulate international assets, \( L_t \), at the fixed interest rate \( i^* \).

The budget constraint presents an equilibrium between expenditures and profits. The expenditure shows the decision which the representative agent makes between consumption and the change in asset position in each period. The profits are defined as the value of production in exportable and
non-traded goods minus the “gross” investment. In this economy all investment takes place in imported goods, produced overseas, and the costs are paid in importable prices.

**Government**

In this model, the Government makes expenditures, \((G_t)\), in domestic goods and pays for them with taxes that it collects from households. The government also has a bond with one period of maturity that pays a nominal interest rate of \(i_t\). The government covers its deficit period by period with the bonds and the lump sum taxes.

\[
P_{o,t} G_t = (B_t - B_{t-1} (1 + i_{t-1})) + \tau P_t C_t + \tilde{r}_{t-1}
\]

(14)

The agents do not received any utility from the government expenditures.

**Country**

The value of net exports are equal to the change in international bond holdings as follows:

\[
P_{e,t} X_t - P_{m,t} \left( M_t + \frac{X_t}{2K_t} I_t^2 \right) = (L_t - L_{t-1} (1 + i_{t-1}^*) )
\]

(15)

where \(X_t\) is the export production and \(M_t\) is the import goods. In aggregate terms export production is identical to the consumption of the export good plus actual exports. Total importable goods equal the consumption of the importable goods. The non-traded sector is in equilibrium between supply and demand by the households and the government.

\[
Y_{a,t} = A_{a,t} K_t^{1-\alpha_e} = C_{x,t} + X_t
\]

(16)

\[
C_{m,t} = M_t
\]

(17)

\[
Y_{n,t} = A_{n,t} = C_{n,t} + G_t
\]

(18)

The usual no-Ponzi game applies to the evolution of real government debt and foreign assets, namely:

\[
\lim_{t \to \infty} B_t \exp (-i_t) = 0 \quad \lim_{t \to \infty} L_t \exp ((-i^* + \Delta s_{t+1}) t) = 0
\]

(19)

This paper imposes more restrictive asset constraints on the international bond by a penalty function if the ratio of debt and traded GDP is violated with a value grater than \(L\). Thus, the solution method avoids a corner solution. The government deficit from the previous period is paid by the lump sum tax,

\[
\frac{|s_{t} L_t|}{Y_{x,t} P_{x,t}} \leq L
\]

(20)

\[
\tilde{r}_t = B_t (1 + i_t)
\]

(21)
• Exchange rate pass-through and stickiness

Prices of exports and imports are determined exogenously for a small open economy. This paper assumes that price changes are incompletely passed through in import prices. In domestic currency these prices are given by:

\[ P_{x,t} = S_t P_{x^*,t} \]  
\[ P_{m,t} = (S_t P_{m^*,t})^\omega P_{m,t-1}^{1-\omega} \]

where \( \omega \) indicates the degree of pass-through of foreign price changes.

The only shocks explored in this paper come from the terms of trade. Specifically:

\[ p_{x^*,t} = p_{x^*,t-1} + \varepsilon_{x^*,t} \quad \varepsilon_{x^*,t} \sim N(0, \sigma^2) \]
\[ p_{m^*,t} = p_{m^*,t-1} + \varepsilon_{m^*,t} \quad \varepsilon_{m^*,t} \sim N(0, \sigma^2) \]

where lower lecture cases denote the logarithm of the respective prices. It is assumed that each price has a unit root process, in logarithms, with random shocks, \( (\varepsilon_{x^*}) \) and \( (\varepsilon_{m^*}) \), independently and identically distributed with standard deviation \( \sigma \).

1.2 Monetary Authority

The central bank practices are consistent with optimal control models. It chooses an optimal interest rate reaction function (equation 25), given its loss function equation, following the linear quadratic regulator problem (equation 27), and its perception of the evolution of the state variables (equation 27).

This study assumes that the monetary authority does not know the exact nature of the private sector model, and “learns” and updates the state space model period by period. The learning process focuses in the evolution of the state variables, \( (x) \), as a function of its own lags as well as the changes in the interest rate (\( \Delta i_t \)),

\[ i_{t+1} = i_t + \Delta i_{t+1} \]  
\[ \Delta i_{t+1} = h (\gamma_j, \gamma_{k+1}) x_t \]  
\[ \max_{\Delta i_t} \Lambda = \lambda_t (x_t - x^*)^2 \]  
\[ x_t = \sum_{j=1}^{k} \gamma_j x_{t-j} + \gamma_{k+1} \Delta i_t + \varepsilon_t \]
The control variable $\Delta \delta_t$ is solved as feedback response to the state variables. The feedback function $h$ is obtained by solving the linear quadratic regulator problem, as discussed in Sargent (1999). The maximization of loss function, $\Lambda$, is done since the weights, $\lambda_t$, are negative.

This paper compares different scenarios of state variables ($x_t$), which are combinations of inflation, growth in Tobin’s $q$ and GDP growth:

\[
x = [\pi_t, \eta_t, \varphi_t]
\]
\[
x^* = [\pi^*, \eta^*, \varphi^*]
\]
\[
\lambda_t = \begin{bmatrix} -\lambda_{1,t} \\ -\lambda_{2,t} \end{bmatrix}
\]

where $\pi_t = \Delta p_t$, $\varphi_t = \Delta q_t$, and $\eta_t = \Delta y_t$ are the logarithm annualized changes in the respective variable.

To do that, the weights on the loss function are chosen to reflect the central bank’s concerns about its policy function, as shown in Table 1:

<table>
<thead>
<tr>
<th>Inflation Target</th>
<th>Inflation and GDP Targets</th>
<th>Inflation and Tobin’s $q$ Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{&lt;\pi^*}$</td>
<td>$\eta_{&lt;\eta^*}$</td>
<td>$\varphi_{&lt;\varphi^*}$</td>
</tr>
<tr>
<td>$\pi_{&gt;\pi^*}$</td>
<td>$\eta_{&gt;\eta^*}$</td>
<td>$\varphi_{&gt;\varphi^*}$</td>
</tr>
</tbody>
</table>

In the first scenario, with a pure inflation target, the policy calls for no change if the key variable is less than the critical value of the policy target. In the other two scenarios, the central bank puts different weights depending the combination of both targets, as Lim and McNelis (2003).

Corresponding to each scenario, the central bank formulates its optimal interest rate feedback rule. It also acts at time $t$ as if its estimated model for the evolution of the variables is true “forever”, that the values of the weights in the loss function are permanently fixed. However central bank “falsifies” its own procedure when it updates the coefficients every period (see Sargent, 1999).
2 Equilibrium and Dynamic Programming Formulations

The competitive equilibrium is defined by stochastic processes \{I_t, K_{t+1}, C_t, P_s, P_m, P_n, L_t, B_t, i_t, \xi_t\}_{t=0}^\infty, where the household optimizes the expected value of consumption (1) subject to the budget constraint (13). The market clears in non-traded sector given by (18), and the restriction on capital is satisfied by (12).

The Lagrangean for solving the model is given by:

\[
\text{Lagr} = E_t \left\{ \sum_{s=0}^{\infty} \xi_s \left[ \frac{C_s^{1-\gamma}}{(1-\gamma)} - q_s [K_{s+1} - (1-\delta)K_s - I_s] + \right. \right.
\]
\[
\left. \left. \lambda_s \left[ \frac{P_s}{P_s} A_{x,s} K_{s}^{1-\alpha_s} - \frac{P_m}{P_s} \left( I_s + \frac{\chi}{2K_s} I_s^2 \right) - \frac{1}{P_s} (B_s - B_{s-1} (1+i_{s-1})) \right. \right. \right.
\]
\[
\left. \left. + \frac{P_m}{P_s} A_{n,s} - \frac{S_s}{P_s} \left( L_s - L_{s-1} (1+i_{s-1}) \right) - C_s (1+\tau) \right] \right\} \right\}
\]

(28)

The variable \( \lambda_t \) is the familiar Lagrangean multiplier representing the marginal utility of wealth. The term \( q_t \), known as Tobin’s \( q \), represents the Lagrange multiplier for the evolution of capital— it is the “shadow price” for new capital.

Maximizing the Lagrangean with respect to \( I_t, K_{t+1}, C_t, L_t, B_t, \lambda_t, q_t \) yields the following first order conditions:

\[ C_t : \xi_t \left[ C_t^{1-\gamma} - \lambda_t (1+\tau) \right] = 0 \]

(29)

\[ L_t : -\xi_t \frac{S_t}{P_t} \lambda_t + E_t \left[ \xi_{t+1} \frac{S_{t+1}}{P_{t+1}} \lambda_{t+1} (1+i_t^*) \right] = 0 \]

(30)

\[ B_t : -\xi_t \frac{\lambda_t}{P_t} + E_t \left[ \xi_{t+1} \frac{\lambda_{t+1}}{P_{t+1}} (1+i_t) \right] = 0 \]

(31)

\[ K_t : E_t \left[ \xi_{t+1} \left( \frac{\lambda_{t+1}}{P_{t+1}} \left( P_{x,t+1} A_{x,t+1} K_t^{1-\alpha_x} (1-\alpha_x) + \right. \right. \right. \right.
\]
\[
\left. \left. \left. P_{m,t+1} \frac{\chi I_{t+1}^2}{2K_t^2} \right) + q_{t+1} (1-\delta) \right] - \xi_t q_t = 0 \]
\]

(32)

\[ I_t : \xi_t \left( q_t - \frac{P_m}{P_t} \lambda_t \left( 1 + \frac{\chi I_t}{K_t} \right) \right) = 0 \]

(33)

\[ \lambda_t : C_t (1+\tau) = \left[ \frac{P_x}{P_t} A_{x,t} K_{t-1}^{1-\alpha_x} + \frac{P_{n,t}}{P_t} A_{n,t} - \frac{P_{m,t}}{P_t} I_{t-1} - \frac{P_m}{P_t} \frac{\chi}{2K_{t-1}} I_{t-1}^2 \right. \]
\[
\left. - \frac{1}{P_t} (B_t - B_{t-1} (1+i_{t-1})) - \frac{S_t}{P_t} \left( L_t - L_{t-1} (1+i_{t-1}) \right) \right] \]
\]

(34)

\[ q_t : K_{t+1} = (1-\delta) K_t + I_t \]

(35)
These equations can be re-expressed as:

\[
C_{t+1}^{-\gamma} = \lambda_t (1 + \tau)
\]  

(36)

\[
S_t \lambda_t = (1 + \bar{C}_t)^{-\beta} \left(1 + i^*_t\right) E_t \left[\frac{P_t}{P_{t+1}^{\lambda_{t+1}}} S_{t+1} \lambda_{t+1}\right]
\]

(37)

\[
\lambda_t = (1 + \bar{C}_t)^{-\beta} \left(1 + i_t\right) E_t \left[\frac{P_t}{P_{t+1}^{\lambda_{t+1}}}\right]
\]

(38)

\[
C_t^{-\gamma} = E_t \left[(1 + \bar{C}_t)^{-\beta} (1 + \tau) (1 + i_t) \frac{P_t}{P_{t+1}^{\lambda_{t+1}}} C_{t+1}^{-\gamma}\right]
\]

(39)

\[
S_t = \frac{\left(1 + i^*_t\right) E_t \left[\frac{P_t}{P_{t+1}^{\lambda_{t+1}}} \lambda_{t+1} S_{t+1}\right]}{(1 + i^*_t) E_t \left[\frac{P_t}{P_{t+1}^{\lambda_{t+1}}}\right]}
\]

(40)

\[
s_t \approx (i^*_t - i_t) + E_t[s_{t+1}]
\]

(41)

\[
q_t = \frac{P_{m,t}}{p_t} \lambda_t \left(1 + \frac{\chi I_t}{K_t}\right) \rightarrow I_t = \left(\frac{P_t}{P_{m,t}^{\lambda_t}} - 1\right) \frac{K_t}{\chi}
\]

(42)

\[
q_t = (1 + \bar{C}_t)^{-\beta} E_t \left[\left(q_{t+1} (1 - \delta) + \frac{\lambda_{t+1}}{P_{t+1}^{\lambda_{t+1}}} \left(P_{x,t+1} A_{x,t+1} K_t^{-\alpha_x} (1 - \alpha_x) + P_{m,t+1} \frac{\chi P_{t+1}^{2}}{2K_{t+1}^2}\right)\right]\right]
\]

(43)

Equation (39) is the typical Euler equation for consumption. The lower case \(s_t\) is the logarithm of the nominal exchange rate \((S_t)\) and \(E_t[s_{t+1}] - s_t\) is the expected rate of exchange rate depreciation.

Equation (43) states that the shadow price of capital equals the marginal cost of investment, including installation costs. The condition can be rewritten as a version of the investment equation posited by Tobin (1971), with the only difference that tradicional Tobin’s \(q_t\), here, is a “nominal” variable, because it is multiplied by marginal value of wealth of the representative agent.

Equation (43) also shows that the solution for \(q_t\) comes from forward-looking stochastic process. As shown by Obstfeld and Rogoff (1996), it states that at an optimum for the household-firm, the date \(t\) shadow price of an extra unit of capital is the discount sum of:

1. The capital’s marginal product next period.

2. The shadow price of capital on the next date \(t + 1\), net of depreciation.

3. The capital’s marginal contribution to lower installation costs next period.

The price of non-traded goods adjusts instantaneously to clear the market for non-traded goods.
Thus the model has three “forward-looking” stochastic Euler equations (39), (41) and (43), which determine $C_t$, $S_t$ and $q_t$. These variables, together with rest of the equations, give the solution for each period with the state of capital and a particular realization of the shocks.

The intra-sectorial consumption decision of the households is solved after the realization of the prices and the total consumption solved intertemporally. The following expressions are the demand for traded, non-traded, importable and exportable goods as functions of aggregate expenditure, real exchange rate and terms of trade:

$$C_{t,t} = \left( \frac{1 - \theta}{\theta} \right)^{\theta-1} Z_t^{\theta-1} C_t$$  \hspace{1cm} (44)

$$C_{n,t} = \left( \frac{1 - \theta}{\theta} \right)^{\theta} Z_t^\theta C_t$$  \hspace{1cm} (45)

$$C_{m,t} = \left( \frac{1 - \alpha}{\alpha} \right)^\alpha J_t^\alpha \left( \frac{1 - \theta}{\theta} \right)^{\theta-1} Z_t^{\theta-1} C_t$$  \hspace{1cm} (46)

$$C_{e,t} = \left( \frac{1 - \alpha}{\alpha} \right)^{\alpha-1} J_t^{\alpha-1} \left( \frac{1 - \theta}{\theta} \right)^{\theta-1} Z_t^{\theta-1} C_t$$  \hspace{1cm} (47)

The equilibrium set of prices in the model is given by the following equations:

$$P_{s,t} = S_t P_{s^*,t}$$  \hspace{1cm} (48)

$$P_{m,t} = (S_t P_{m^*,t})^\omega P_{m,t-1}^{1-\omega}$$  \hspace{1cm} (49)

$$P_{r,t} = \frac{(1 - \alpha)^{\alpha-1}}{\alpha^\alpha} P_{x,t}^{1-\alpha} P_{m,t}^{1-\alpha}$$  \hspace{1cm} (50)

$$P_{n,t} = \frac{P_{r,t}}{Z_t}$$  \hspace{1cm} (51)

$$P_t = \frac{(1 - \theta)^{\theta-1}}{\theta^\theta} P_{r,t}^\theta P_{n,t}^{1-\theta}$$  \hspace{1cm} (52)

2.1 Parameterized Expectations Algorithm

This subsection studies the parameterized expectations approach to this model. Following Marcet (1988, 1993), Den Haan and Marcet (1990, 1994), and Duffy and McNelis (2001), the approach of this study is to “parameterize” the forward-looking expectation in this model, with non-linear functional forms $\psi_C$, $\psi_S$, and $\psi_q$. 

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Consumption and nominal exchange rate come from expectation equations (39 and 41), and are transformed as “parametrized” equations as follow:

\[ C_t = \psi_{ct}(z_{t-1}; \Omega_c) \]  
\[ S_t = \psi_{st}(z_{t-1}; \Omega_s) \]  
\[ z_{t-1} = \{K_t/K^*, L_{t-1}, i_t/i^*, P_x^*, t, P_f^*, t\} \]

The symbol \( z_{t-1} \) represents a vector of observable “instrument” variables known at time \( t \). These “state” variables are: the ratio of initial capital at \( t \), \((k_t)\), to its steady state value, \((K^*)\), international bonds from the last period, \((L_{t-1})\), the ratio of the domestic nominal interest rate, \((i_t)\), to the international rate, \((i^*)\), and the realization of the international prices of exports, \((P_x^*, t)\), and imports, \((P_f^*, t)\).

The symbol \( \Omega_c \) and \( \Omega_s \) represent the parameters for the expectation function, while \( \psi_c \) and \( \psi_s \) are the expectation approximation functions.

Combining equations (42) and (43) find investment is a “forward looking” variable, and is parameterized in the following way:

\[ I_t = E_t \left[ \frac{q_{t+1} (1 - \delta) P_t}{\lambda_t (1 + C_t)^{\beta} P_{m,t}} + \frac{\lambda_{t+1} P_t}{\lambda_t (1 + C_t)^{\beta} P_{t+1}} \left( \frac{P_{x,t+1}}{P_{m,t}} A_{x,t+1} K_t^{-\alpha_x} (1 - \alpha_x) + \frac{P_{m,t+1}}{2K_{t+1}} \frac{\chi I_{t+1}^2}{\chi} \right) - 1 \right] \]  
\[ I_t = \psi_{it}(z_{t-1}; \Omega_i) \]

This change has two advantages. First, it places a limitation on the “domain of the expectations”, since the key problem is to find a good set of parameters that will not give a “far away” solution. A small deviation on \( q_t \) implies a large deviation in \( I_t \), it however, produces smaller deviations in \( q_t \), so using expectations on \( q_t \) is less efficient. Secondly, working with this variable gives the remaining equations have close form solutions. The combination of these advantages thus gives more accurate and computationally faster solutions.

The functional form for \( \psi_t \) is usually second-order polynomial expansions (see, for example, Den Haan and Marcet, 1994, Schmitt-Groh and Uribe, 2002). However, Duffy and McNelis (2001) have shown that neural networks have produced results with greater accuracy for the same number of parameters, or equal accuracy with fewer parameters, than the second-order polynomial approximation.

The model was simulated for repeated parameter values for \( \Omega_c, \Omega_s \), and \( \Omega_i \) until convergence was obtained for the expectation errors and the algorithm found the optimal coefficients \( \Omega_c, \Omega_s, \) and \( \Omega_i \).

In the algorithm, the following non-negativity constraints for consumption and the stocks of capital for next period were imposed:
The section discusses the calibration of parameters, initial conditions, and stochastic processes for the exogenous variables of the model.

### 3.1 Parameters

The parameter settings for the models appear in Table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Consumption</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>1.500</td>
<td>0.500</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.500</td>
<td></td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.500</td>
<td>0.006</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.500</td>
<td>0.023</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.028</td>
<td></td>
</tr>
<tr>
<td>( \chi )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Others</td>
<td>0.047</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Many of the parameter selections are common in the literature. The constant relative risk aversion is set at 1.5. The standard deviations of the shocks, \( \sigma \), on the foreign prices are set at 0.01. The shares in the traded sector, for exportable goods and in production of exportable good, are equal to 0.5. The coefficient \( \beta \), following Mendoza (1995), is set at 0.009.

The adjustment cost scale and depreciation rate are at 0.028 and 0.023 respectively. The tax is 10% of the consumption.

This study also uses the following initial conditions: initial bonds domestic and foreign are zeros. Non-traded prices, nominal exchange rate and foreign prices are normalized to 1.

The value of the international interest rate is fixed at 4 percent in annual terms for that in quarterly data it is one percent.

### 3.2 Steady State

As usual in General equilibrium models the initial conditions are obtained by the value of steady state of the variables and value of the parameters. In this model, they are as follows:

\[
P_x = 1 \\
P_m = 1
\]
\[ J = \frac{P_x}{P_m} = 1 \quad (62) \]

\[ P_t = \frac{(1 - \alpha)^{\alpha - 1}}{\alpha^\alpha} \quad (63) \]

\[ Z = P_t = \frac{(1 - \alpha)^{\alpha - 1}}{\alpha^\alpha} \quad (64) \]

\[ P = \frac{(1 - \theta)^{\theta - 1}}{\theta^\theta} \left( \frac{(1 - \alpha)^{\alpha - 1}}{\alpha^\alpha} \right)^\theta \quad (65) \]

\[ i = i^* \quad (66) \]

\[ \tilde{C} = (1 + i^*)^{\frac{1}{\delta}} - 1 \quad (67) \]

\[ \lambda = \frac{C^{-\gamma}}{1 + \tau} \quad (68) \]

\[ q = \frac{(1 + \delta \chi)}{\lambda} \quad (69) \]

\[ G = \tau PC - B i^* \quad (70) \]

\[ C_x = \left( \frac{1 - \alpha}{\alpha} \right)^{\alpha - 1} \left( \frac{1 - \theta}{\theta} \right)^{\theta - 1} Z^\theta C \quad (71) \]

\[ C_m = \left( \frac{1 - \alpha}{\alpha} \right)^{\alpha} \left( \frac{1 - \theta}{\theta} \right)^{\theta - 1} Z^{\theta - 1} C \quad (72) \]

\[ C_n = \left( \frac{1 - \theta}{\theta} \right)^{\theta} Z^{\theta} C \quad (73) \]

\[ M = C_m \quad (74) \]

\[ X = M + \delta K + \frac{\chi \delta^2}{2} K \quad (75) \]

\[ Y_n = C_n + G \quad (76) \]

\[ A_n = Y_n \quad (77) \]

\[ I = \delta K \quad (78) \]

\[ K = \frac{(C_x + M)}{\left( \frac{(i^* + \delta)(1 + \delta \chi) - \frac{\chi \delta^2}{2}}{1 - \alpha \delta} - \delta - \frac{\chi \delta^2}{2} \right)} \quad (79) \]
4 SIMULATION ANALYSIS

The aim of the simulations is to analyze which is the best policy that the central bank should follow under different economy conditions, “scenarios”. Three central bank policies are analyzed in this study: pure inflation, \((\pi)\), inflation and growth, \((\pi, \eta)\), and inflation and Tobin’s \(q\), \((\pi, \varphi)\).

The comparison is made in four scenarios. First, the central bank needs to learn the private sector behavior and there is a full pass-through of the exchange rate, “Learning, with \(\omega = 1\)”. Second, central bank “learns” private sector behavior and there is a low pass-through “Learning, with \(\omega = 0.1\)”. Third, the monetary authority follows an optimal Taylor rule with full pass-through of the exchange rate, “Optimal policy, with \(\omega = 0.1\)”. The fourth has an optimal Taylor rule with low pass-through, “Optimal policy, with \(\omega = 0.1\)”. The values of the pass-through are extreme but for the purpose of the comparison they are indicative.

The number of simulations, for each scenario and each policy, is 1500 with each containing a time-series of 350 observations.

\[
A_x = \frac{C_x + M + I + \delta x^2 K}{K^{1-\alpha_x}}
\]

\[
Y_x = A_x K^{1-\alpha_x}
\]

**Figure 1:**

Two realizations (1,2) of the series of Terms of Trade (tot) and Consumption (ct) under low pass-through \((\omega=0.1)\) and an optimal Taylor rule.
Figure 1 presents two random realizations of the terms of trade, and the optimal consumption paths under different policies in an economy with Taylor rules and low pass-through exchange rate. The random walk process followed for international prices produces large fluctuations in terms of trade price. It creates large differences in the realizations of the series.

Table 3 presents the mean and standard deviation of these series and their mean after 100 simulations. It shows the large difference between each realization compared with the steady state value for all scenarios (2.02).

The second draw yields a better value than in the steady state. If that is the realization of the economy, people are better with the shock than in the steady state. Thus, it is not helpful to compare the welfare effect with a “steady state” benchmark. To avoid this problem comparison are done only between policies scenarios.

Table 4 presents the value of mean and standard deviation of the sum of welfare function, for the three policies in the four scenarios. The analytical steady state welfare, by comparison, is -142.12. When it is computed with the same number of observations than the sample its value is -135.0034.

Table 5:
I use a different approach for comparison. For each realization of the shocks the model is solved for the 12 different situations (four scenarios and three policies). I then compare welfare effects of the central bank policies. The comparisons are in groups of two policies: inflation, ($\pi$), versus inflation and growth, ($\pi, \eta$); inflation, ($\pi$), versus inflation and Tobin’s $q$, ($\phi, \pi$); inflation and growth, ($\pi, \eta$), versus inflation and Tobin’s $q$, ($\phi, \pi$). The left side of Table 5 presents the percentage (with 1500 simulations) of each chosen policy. The table was computed with the maximum value between two numbers, the welfare value for each policy. The right side of this table presents the loss if the central bank does not follow the policies used in the first column. These values are expressed in terms of “percentage of loss” based on the mean of consumption.

For example, for the first scenario "Learning, with $\omega = 1$", inflation policy was chosen 1396 times (93.1%) versus inflation and growth that was chosen only 104 times (6.9%) from 1500 random realizations. If it is used the right side of the table, the loss in the consumption is 0.63% when the central bank moves from inflation target to inflation and growth targets.

The proportions are robust also when they are filtered by removing the "wrong" or "outlier" simulations, in the sense that if a simulation violated the ratio of international bonds allowed in the model or if the simulation goes to the 5% level of the Marcet statistic (1994), it is removed.

Figure 2:

It is also possible to compare the reaction of the representative agent when the agent faces different scenarios even with the central bank following the same policy rule. Figure 2 shows the optimal paths of consumption in different scenarios for one particular realization under the inflation target regime. It shows large fluctuations when the central bank follows a “learning” process than if it follows an optimal Taylor rule. The low pass-through, as expected, gives less welfare than full pass-through economy.
The results give support for a pure inflation targeting regime. The only exception is for the case with a lot “noise” in the economy, that is with “sticky” information in the learning process and low pass-through. Then it is preferable to use a second variable. In this case the second variable is growth of output, not the growth of Tobin’s $q$.

5 CONCLUSION

This paper uses a stochastic dynamic general equilibrium model for small open economy, with terms of trade shocks and different degrees of information stickiness. This paper has produced some evidence that the central bank should target another variable (besides inflation target) to achieve better welfare.

The argument is on the case where there is incomplete pass-through of exchange rate and the monetary authority faces some stickiness in “learning” the true model. The combination of these two sources of “errors” for the central bank, gives trade-off for policy stances. This result is consistent with recent results in the literature of “new neoclassical synthesis” in macroeconomics (see Canzoneri, Cumby and Diba, 2002). However this model has neither labor decisions nor productivity shocks. This model shows that if there are more sources of “noise” in the economy, a second variable in the policy function does improve welfare.

There is a clear extension: if there is a productivity shock in the model, will pure inflation target continue to be the optimal policy?

Bibliography


