Why Do Governments Choose Different debt Maturity Structures?*

Darío Cestau**

Abstract

This paper uncovers a new stylized fact: Debt maturity structure is non-monotonic in income. Countries with low and high levels of income tend to issue (proportionally) more short term debt than middle income economies. The existing literature on debt maturity does not offer an explanation for this new observation. I develop a model that predicts this behavior. In this model governments issue long term debt to reduce their exposure to income shocks that affect future financing costs. In equilibrium risk averse bondholders charge a premium to hold this debt. First, I show that the premium increases with long term debt levels. Then I show that the increase is faster for low income economies. A country issuing long term debt above the optimal level may drain its resources on too expensive debt, increasing the probability of a financial crisis in case a bad shock occurs. Governments of high income countries face financial crises with low probability and thus prefer the cheaper short term debt. Governments of middle income and poor economies, who are more likely to have financial crises, choose longer maturities to cover against this risk. Poor economies are unable to issue high levels of long term debt because they face expensive premiums.

Resumen

Este artículo revela un nuevo hecho estilizado: la estructura de maduración de deuda es no-monotónica en el ingreso. Países con bajos y altos niveles de ingreso tienden a emitir (proporcionalmente) más deuda de corto plazo que economías de ingreso medio. La literatura existente sobre maduración de deuda no ofrece una explicación a esta nueva observación. El presente artículo desarrolla un modelo que predice este comportamiento. En el modelo, los gobiernos emiten deuda de largo plazo para reducir su exposición ante shocks sobre el ingreso, que afectan los costos de financiamiento futuro. En equilibrio, los tenedores de bonos adversos al riesgo cargan un premio por mantener esta deuda. Primero, se muestra que el premio aumenta con niveles de deuda de largo plazo. Luego, se muestra que el incremento es más rápido para economías de bajos ingresos. Un país que emite deuda de largo plazo por encima del nivel óptimo puede ver drenados sus recursos con deuda muy costosa, aumentando la probabilidad de una crisis financiera en caso de que ocurra un shock malo. Los gobiernos de países de ingresos altos se enfrentan a crisis financieras con baja probabilidad y,

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en consecuencia, prefieren la deuda mas barata, aquella de corto plazo. Los gobiernos de países de ingresos medios y bajos, los cuales es más probable enfrenten crisis financieras, eligen maduraciones más largas para cubrirse frente a este riesgo. Los países pobres se encuentran imposibilitados de emitir altos niveles de deuda de largo plazo debido a que se enfrentan con premios altamente costosos.

1. **Introduction**

This paper analyzes the choice of sovereign debt maturity structure. I present empirical evidence of a non-monotonic relationship between income per capita and the government’s choice of debt maturity. Data reveals that governments of high income and low income economies have higher proportions of short term debt than those of medium income economies.\(^1\) I develop a model that explains the cross-country differences in the maturity structure of public debt and why we observe a non-monotonic relationship (“U-Shape”) between the short term share of total debt and income per capita.

The object of study is puzzling because classical models predict low income countries issue long term debt. Thus many researchers have attempted to explain why do they issue too much short term debt. There is literature on emerging economies like Calvo (1988), Blanchard and Missale (1994), Barro (1995, 1997), Chari and Kehoe (2003), and Tirole (2002, 2003), that states that short term debt is a consequence of market inefficiencies caused by the lack of control and enforcement of debt contracts. Governments have incentives to raise inflation in order to lower the real value of public debt. These authors find that a short maturity structure alleviates the existing moral hazard problem on the debtor side. Some authors like Rodrik and Velasco (1999) and Jeanne (2004) argue that shorter maturities lower the incentives of governments to strategically default on debt. Moreover they add that a short term structure provides incentives to introduce fiscal reforms. Broner, Lorenzoni and Schmukler (2006), state that what drives emerging economies to move toward shorter maturities are the lenders’ increase in risk aversion during financial crisis.

This paper presents a new and different approach to solve this puzzle. I reformulate the problem to ask why do they issue more short term debt than middle income economies. I do not draw on market failures. Instead I argue that differences in maturity structure come from country risk heterogeneity and bondholders risk aversion. Governments issue long term debt to reduce their exposure to income shocks that affect future financing costs. Lenders holding long term debt (generally) turn to the secondary market to liquidate their assets before maturity. Governments’ income shocks affect the price at which they can sale their assets and therefore bondholders charge a premium for facing this price risk. Their risk exposure increases with their holdings of long term debt. Hence they demand higher premiums. I also find that the volatility of income shocks is larger on average for low income economies. Thus, lenders holding long term debt from them demand higher premiums than they would do for middle income economies. As a consequence, low income countries issuing long term debt in similar proportions to middle income ones, would face too expensive premiums. This would drain their resources making the country vulnerable to a financial crisis in case a bad shock occurs. As an optimal response, poor economies decrease their long term debt levels by issuing more short term debt.

In the model the economy lasts 3 periods (dated by period 0, period 1 and period 2). In period 0 the Government needs to rollover an exogenous initial debt coming to maturity. It can choose from a combination of short term debt with maturity in period 1, and long term debt with maturity in period 2. In period 1 there is a stochastic shock on the government’s expected future income. Similar from what we observed in data, I assume that the shock volatility decreases with government’s income. The shock affects the government’s capability of rolling over (with new debt issuance) short term debt maturing in this period. It also affects long term bonds’ prices in the secondary market. Period 0 Lenders holding long term debt, which live 1 period, liquidate their assets before maturity

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1. To my knowledge, there is no precedent work showing empirical evidence on this behavior.
and thus face a price risk on these bonds. Thus, they require a premium for carrying this risk. Period 1 bondholders optimally decide how much debt they want to hold for next period. In case they failed to provide enough funding the government enters a financial crisis which is costly.

The model has the following implications. When the expected income is high and the volatility low, governments will have sufficient resources to rollover debt in period 1, regardless the realization of the shock. Therefore it can absorb the price risk faced by bondholders with no cost, by issuing short term debt (as classical models predict). This is the case of high income economies. In case of lower expected resources and higher volatility, governments may have insufficient funds to rollover debt in period one. The model predicts it is optimal for the government to move toward longer maturities in order to decrease the expected loss from a rollover crisis. This is the case of middle income and low income economies. As the government’s resources become smaller and the variance larger, bondholders require higher risk premiums for holding long term debt. High premiums lower the government’s remaining resources to rollover short term debt maturing in period 1 (because more resources are used to repay long term debt). Therefore, there is an increase in the probability of a rollover crisis, as opposed to a decrease in the potential level of it. For sufficiently high risk premiums and long term debt levels, a higher probability of financial crisis will offset the benefits from a long maturity structure. Therefore, it is optimal to move towards a shorter maturity structure.

As a result, governments of high income economies prefer to hold the risk by choosing a short term maturity structure. Governments of middle income economies transfer the risk to bondholders by issuing long term debt. Governments of low income and unstable economies prefer to hold part of the risk by choosing a shorter maturity structure. All in all, the model predicts optimal debt maturity structures which show the same pattern that is observed in the data.

The rest of the paper is organized as follows. Section 2 shows the empirical results. Section 3 presents the model. Section 4 describes its implications and presents a simple example. Section 5 presents some simulation results. Section 6 concludes this work.

2. Stylized Facts

This section studies the relationship between a country’s income per capita and the debt maturity structure chosen by its government. In order to characterize the debt maturity structure, I use the ratio between government short term debt and total debt. I measure this relationship for three definitions of short term debt: Government’s debt maturing within 1 year, 3 years, and 5 years respectively. The data reveals a non-monotonic relationship between governments’ debt maturity choice and income per capita. The governments of low income and high income economies possess a higher proportion of short term debt than those of middle income economies.

This paper considers debt to be the bonds and borrowings of a government (trade credits, stocks, and others are excluded from the analysis). Data on this type of debt was collected from Bloomberg. For the year 2009 there is data available for 97 countries, disaggregated by year of maturity. There is no disaggregated information for previous years. Income per capita and GDP were collected from the World Bank. The last information available is for 2007.

2.1. Existence of a “U-Shaped” relationship

In order to test the existence of the non-monotonic relationship described above, I study the governments debt maturity structure for 4 income categories. Using World Bank Classification I group countries into: low income (from $0 to $800), lower middle-income ($800 to $3,300), higher middle-income ($3,300 to $10,000), and high income (more than $10,000) economies. Table 1 shows that governments of low incomes and high income economies have larger shares of short term debt than higher medium-income economies.
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Table 1: Short term to total debt ratio: by World Bank Income Group

<table>
<thead>
<tr>
<th>Classification</th>
<th>1 Year</th>
<th>3 Years</th>
<th>5 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Income</td>
<td>52.0%</td>
<td>81.5%</td>
<td>88.3%</td>
</tr>
<tr>
<td>Lower Middle Income</td>
<td>35.4%</td>
<td>58.7%</td>
<td>68.2%</td>
</tr>
<tr>
<td>Upper Middle Income</td>
<td>18.5%</td>
<td>40.4%</td>
<td>54.2%</td>
</tr>
<tr>
<td>High Income</td>
<td>25.0%</td>
<td>47.1%</td>
<td>61.5%</td>
</tr>
</tbody>
</table>

The same analysis is carried on dividing countries into quintiles and deciles according to income per capita. Results exhibit the same pattern shown before. Shares of short term debt decrease when income per capita increases, to then increase again. This result holds again for the three definitions of short term debt. Figure 1.1 and 1.2 present these results.

Results indicate that there is a non-monotonic relationship between income per capita and debt maturity structure: the governments of high and low income economies rely more on short term debt than medium income ones. This non-monotonic relationship is stronger for the 1 year short term definition (the accounting definition of short term debt). Differences diminish for longer maturities.
2.2 Functional Form

Given the previous results, this section tests the fit of the following quadratic functional form:

\[ \frac{S}{T_i} = \beta_0 + \beta_1 Y_i + \beta_2 Y_i^2 + \varepsilon \]

\( S/T_i \) is the share of short term to total debt, \( Y \) is income per capita, and \( Y^2 \) is income per capita squared. \( \beta_j \) for \( j = 0, 1, 2 \) stands for the corresponding coefficients and \( \varepsilon \) for the error term. This is a regression of the share of short term debt to total debt on income per capita and income per capita squared. I separately perform the regression for two groups of countries: developed and developing nations. My model predicts a structural difference between both groups and I want to study if this is satisfied in the data. The criterion used to divide the sample was taken from the IMF.

Results show different functional forms for both groups. Coefficients on income per capita and income per capita squared are statistically significant for the developing world according to the three definitions of short term debt. For developed nations, both coefficients are insignificant for all definitions of short term debt. As my model predicts, countries behave similarly until they fall under an income threshold, where debt maturity starts depending on income. Long term debt increases as we move away from that threshold but it decreases again once income get far away from that threshold. The derived functional form for developing countries shows the same features that my model. It is non-monotonic on income. Results are presented in tables 2 and 3 and figures 2 and 3 respectively.

**Table 2: Regression Results: developing nations**

<table>
<thead>
<tr>
<th></th>
<th>1 Year</th>
<th>3 Years</th>
<th>5 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>-8.79</td>
<td>-10.15</td>
<td>-7.72</td>
</tr>
<tr>
<td></td>
<td>(2.27)</td>
<td>(2.85)</td>
<td>(2.56)</td>
</tr>
<tr>
<td>Income²</td>
<td>4.99</td>
<td>5.74</td>
<td>4.17</td>
</tr>
<tr>
<td></td>
<td>(1.67)</td>
<td>(1.89)</td>
<td>(1.88)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.51</td>
<td>0.78</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>R²</td>
<td>0.23</td>
<td>0.23</td>
<td>0.17</td>
</tr>
</tbody>
</table>

**Table 3: Regression Results: developed nations**

<table>
<thead>
<tr>
<th></th>
<th>1 Year</th>
<th>3 Years</th>
<th>5 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>-9.50</td>
<td>-1.24</td>
<td>-1.51</td>
</tr>
<tr>
<td></td>
<td>(6.9)</td>
<td>(6.19)</td>
<td>(6.26)</td>
</tr>
<tr>
<td>Income²</td>
<td>1.06</td>
<td>2.36</td>
<td>2.71</td>
</tr>
<tr>
<td></td>
<td>(8.21)</td>
<td>(8.41)</td>
<td>(7.44)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.20</td>
<td>0.46</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.14)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>R²</td>
<td>0.10</td>
<td>0.04</td>
<td>0.03</td>
</tr>
</tbody>
</table>

To sum up, results indicate that there is a non-monotonic relationship between income per capita and debt maturity structure: the governments of high and low income economies rely more on short term debt than middle income ones. This relationship can be explained with a quadratic functional form among variables for the case of the non-developed world. Regressions reveal statistically significant coefficients under all scenarios and a high R-squared compared to the literature benchmark. For developed countries the share of short term debt appears to be constant and higher than for middle income economies.
Figure 2: Short term to total debt ratio: developing nations
Next section will show how this functional form can be explained as the outcome of a government and bondholder optimization problem.

3. The Model

The economies are modeled like in Broner, Lorenzoni and Schmukler (WP 2006). This paper models a government which has to rollover maturing debt. It has an exogenously given expected income, and
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an exogenous given variability in expected income. The model presented in this section differs from Broner, Lorenzoni and Schmukler (WP 2006) in the following: here it is assumed that the governments of low income economies are more unstable in the sense that they have greater variance in expected income than the governments of high income economies. Empirical evidence supporting this assumption can be found on the Appendix.

Broner, Lorenzoni and Schmukler (WP 2006) also assume that bondholders preferences exhibit absolute decreasing risk aversion, where risk aversion is determined by the lender’s wealth. While in this model there is no need for such an assumption.

The main difference between Broner, Lorenzoni and Schmukler (WP 2006) and this paper is that the first one studies whether a crisis is caused by a reduction in the government’s expected income or an increase in bondholders risk aversion. This paper focuses on studying the optimal debt maturity choice of high income, middle income, and low income economies.

**Debtor Country**

The economy lasts three periods: Period 0, period 1 and period 2. On period 0 governments must roll-over debt coming to maturity. They may choose between 1-period bonds (short term) and 2-period bonds (long term). One-period bonds are bond maturing in period 1 and 2-period bonds are bonds maturing on period 2. For simplifying purposes governments are assumed to be risk neutral.

Period 0 budget constraint is:

\[ D_0 = p_s D_S + p_L D_L \]  

(1)

\( D_0 \) is debt coming to maturity. \( D_S \) and \( D_L \) are the amount of 1-period bonds (short term) and 2-period bonds (long term) issued on date 0 at prices \( p_s \) and \( p_L \) respectively.

On period 1, the government finances its stock of short term debt issued on period 0, with a new bond issuance \( D_{S1} \). When \( D_{S1} \) is insufficient to cover \( D_S \) that is, the government is unable to borrow in order to rollover debt, it makes a fiscal adjustment to cover the difference. As in Broner, Lorenzoni and Schmukler (WP 2006) fiscal adjustment costs exceeds the earnings from it. Here it is assumed that sudden budget cuts or tax raises have an associated cost. Governments unable to rollover debt face a rollover crisis. The size of this crisis is equal to the difference between period 0 short term debt and period 1 short term debt.

\[ D_S = x + p_{S1} D_{S1} \]  

(2)

Where \( p_{S1} \) is the price of short term bond issuance, \( x \) is the fiscal adjustment and \( C'(x) \) is the cost of the fiscal adjustment. As in Broner, Lorenzoni and Schmukler (WP 2006), \( C'(x) \) is assumed to be a strictly convex function.

On period 2 the government receives an income inflow \( \hat{Y} \). This income inflow can take values \( Y \) when the economy is in the good state of nature or 0 if it is in the bad state of nature. If \( \hat{Y} = Y \) the government can repay debt and use the surplus for other government activities that give him utility. The government’s utility function then is:

\[ W = E_0[-C(x) + \max\{\hat{Y} - D_L - D_{S1}, 0\}] \]  

(3)

Period 0 expected income is \( E_0[\hat{Y}] = \pi_0 Y \), where \( \pi_0 \) is a given probability of being in the good state of nature. Period 1 expected income is \( E_1[\hat{Y}] = \pi Y \) where \( \pi \) is period 1 updated.
probability of being in the good state of nature. \( \pi \) is a stochastic variable distributed on the interval \([\pi, \bar{\pi}]\) and its expected value is \( E[\pi] = \pi_0 \). The variance of \( \pi \), \( V[\pi] \), increases as \( \pi_0 \) becomes smaller.

**Investors**

There are two 2-living period overlapping generations of investors: the ones who are born in period 0 and consume in period 1, and those who are born in period 1 and consume in period 2.

Date 0 investors want to maximize date 0 expected utility function \( E_0[u(c)] \), where \( u(\cdot) \) is increasing and concave (agents are risk averse). They can invest their initial wealth \( w \) in an international risk-less asset bond (offered at price 1), and short-term bonds and long term bonds offered by the government. They have to liquidate their outstanding holdings of long term bonds on date 1.

Their budget constraints are:

\[
\begin{align*}
w &= b + p_S d_S + p_L d_L \\
c &= b + d_S + p_{L,1} d_L
\end{align*}
\]

where \( b \) is international asset holdings and \( d_S \) and \( d_L \) are government’s short term and long term bonds holdings. In period 1, period 0 short term debt is paid with certainty (there is no default premium) and long term debt is liquidated at price \( p_{L,1} \).

Date 1 investor’s portfolio is a compound of the international risk-less asset, period 1 issuance of short-term bonds, and date 0 long-term bonds liquidated by the previous investor generation, with remaining maturity of one period. Like in Broner, Lorenzoni and Schmukler (WP 2006), it is assumed, for simplicity, that period 1 investors are risk neutral.

An equilibrium in this economy is a set of prices \( \{p_S, p_L, p_{S,1}, p_{L,1}\} \) and quantities \( \{D_L, D_S, D_{S,1}\} \) such that the government and bondholders maximize their utility at given prices.

**Bond prices**

As a result of period 1 investor’s risk neutrality, period 1 bond prices are equal to the probability of repayment \( \pi \): \( p_{L,1} = p_{S,1} = \pi \). As \( \pi \) is stochastic, secondary market long term bond’s price is variable. Then, period 0 bondholders face a price risk on long term bonds.

Using period 1 bond prices, market clearing conditions \( d_S = D_S, d_L = D_L \) and period 0 lender’s first order conditions, period 0 bond prices can be derived.

Period 0 first order conditions are:

\[
\begin{align*}
E[u'(w + (1 - p_S)D_S + (\pi - p_L)D_L)(\pi - p_L)] &= 0 \\
1 - p_S &= 0
\end{align*}
\]

Substituting the second into the first equation it is obtained that:

\[
E[u'(w + (\pi - p_L)D_L)(\pi - p_L)] = 0
\]

which implicitly defines \( p_L(D_L) \), the price for long term bonds as a function of \( D_L \).
4. Implications

The following Lemma is derived from bondholder’s first order condition:

- **Lemma 1**: If long term debt is positive, \( D_L > 0 \), the price of long term is \( p(D_L) < 0 \). If \( D_L = 0 \), then the price of long term debt is \( p(D_L) = \pi_0 \). [Proof in Appendix]

The lemma implies that bondholders require an expected gain from liquidating their holdings of long term debt in period 1. The intuition behind this result is lenders must be compensated for the risk faced for holding long term debt. Consequently long term debt is more expensive than short term debt for the government.

- **Lemma 2**: The price of long term debt \( p(D_L) \) is decreasing in \( D_L \). [Proof in Appendix]

This Lemma says that whenever the government issues more long term debt the cost of it will increase. The risk premium is higher for higher shares of longer maturities. As bondholders are exposed to more risk they require a bigger compensation for holding such risk. We should also expect that for a mean persevering spread in \( \pi \) the slope of \( p(D_L) \) decreases (prices are even more decreasing in long term debt). If bondholders are more risk averse they will require a bigger compensation for holding risk.

Then, an increase in long term debt has two effects on the expected fiscal adjustment cost in period 1. First, it lowers the maximum possible level of fiscal adjustment. Second, it lowers the government’s total resources in period 2 because more income is devoted to repay long term debt, increasing the probability of incurring in a rollover crisis in period 1 (The amount period 1 bondholders are willing to lend decreases when government resources decrease). When the effect of \( D_L \) on long term bond prices (lemma 2) rises, the second effect increases while the first one decreases.

Consequently there are three possible scenarios:

1. A Government with resources satisfying \( \pi Y - D_0 > 0 \): This government will not face rollover crisis in period 1. The optimal amount of long term debt issued by this government is \( D_L = 0 \). Any positive level of long term debt is suboptimal because it increases the borrowing cost.

2. A Government with resources satisfying \( \pi Y - D_0 < 0 \) and first effect dominating over second effect: This government faces a rollover risk. An increase of long term debt will lower the expected cost from a rollover crisis.

3. A Government with resources satisfying \( \pi Y - D_0 < 0 \) and second effect dominating over first effect: This government faces a rollover risk. A decrease of long term debt will lower the expected cost from a rollover crisis.

Next section presents an example where wealthy economies are located in scenario 1. Middle income economies are located in scenario 2, and lower income economies in the third scenario. As a result, governments of high income economies and low income economies have larger shares of short term debt than governments of middle income economies.

5. Simulating the model

The following is an exercise showing how the costs and benefits from long term debt can interact to give a U-shape maturity structure.
Governments must refinance debt coming to maturity $D_0 = 50$. The income perceived in the good state of nature is $Y = 100$. There is a continuum of governments which can take values of $\pi_0$ between $[.5, 1]$ (therefore they are not initially in default). In order to obtain closed form solution for $p_L(D_L)$, $\pi$ is assumed to be uniformly distributed on $[\pi_0 - (1 - \pi_0), 1]$ and $C(x) = x^2$. Date 0 Investor Utility function is $U(C) = -exp[-\delta C]$.

From equation 8 it is obtained:

$$p_L(D_L) = \frac{e^{D_L H \delta} - e^{D_L L \delta} - D_L H \delta e^{D_L L \delta} + D_L L \delta e^{D_L H \delta}}{D_L (e^{D_L H \delta} - e^{D_L L \delta})} \quad (9)$$

Government must choose $D_L, D_S, D_{S1}$ and $x$ to maximize period 0 objective function:

$$W = E_0[-C(x) + \pi[Y - D_L - D_{S1}]]$$

s.t.

$$D_0 = D_S + p_L D_L$$

$$x = D_S - D_{S1}, 1$$

$$p_L(D_L)$$

This problem can be solved backwards. In period 1 the government must choose $D_{S1}$ and $x$ to maximize:

$$E_1[-C(x) + \pi[Y - D_L - D_{S1}]]$$

s.t.

$$x = D_S - D_{S1}$$

As a result the Government increases its period 1 utility function if he minimizes its fiscal adjustment. Then $x = max\{D_0 - \pi Y + (\pi - p_L) \times D_L, 0\}$. Thus period 0 government’s maximization problem is:

$$\max_{D_L} W(D_L) = 100\pi_0 - D_0 - (\pi_0 - p_L(D_L)) D_L + \int_{x(\pi, D_L) \geq 0} [x(\pi, D_L) - C(x(\pi, D_L))] dF(\pi)$$

This problem has no closed form solution; it has to be solved numerically. Next subsection presents the optimal shares of short term debt using different values for delta.

5.1 Simulation

The example is simulated for $\delta = [0.05, 0.075, 0.1]$. It illustrates how an optimization problem between risk adverse lenders and governments facing rollover risk, can result in a U-Shape debt maturity structure, where short term debt shares are larger for high income and low income economies than for middle income economies.
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Figure 4: Debt Maturity Structure: $\delta = 0.05$

Figure 5: Debt Maturity Structure: $\delta = 0.075$
The simulations reproduce the empirical behavior shown in the data. We can see how countries facing the highest income have a short term debt structure. Furthermore, as shown in section 2, this model also reproduces the fact that their maturity choice does not differ from one another. Figures 4, 5, and 6 reveal countries with income per capita between $20,000 and $80,000 have similar debt structures. When the income decreases, the government’s optimal debt structure moves toward a longer maturing structure. As income continues to decrease it is optimal to shift toward a shorter maturity structure once again.

All in all, the model reproduces two facts observed in data: governments of high income and low income economies have higher proportions of short term debt than those of medium income economies, and governments of high income have similar debt maturity structure.

5.2 Analysis of Results

Government Utility function can be separated into two parts. The first part, $A = 100\pi_0 - D_0 - (\pi_0 - p_L(D_L))D_L$, reflects the expected earnings in period 2. The second part, $B = \int_{x(\pi, D_L) \geq 0} [x(\pi, D_L) - C(x(\pi, D_L))]dF(\pi)$, reflects the expected rolling over cost in period 1.

Long term debt affects Government utility function via two ways. From lemma 1 and lemma 2 we know long term debt increases bond premiums, which augment the financing cost of the initial debt $D_0$. The extra cost is reflected in “A”; period 2 expected earnings decreases with long term debt. The second way it affects government utility is through the expected period 1 rolling over cost “B”. Long term debt has an ambiguous effect on this term. More long term debt means less debt maturing in period 1, thus a smaller potential financial crisis. On the other hand, as period 2 resources decreases, governments may be unable to find lenders in period 1, entering a costly financial crisis.

I define cost and benefit of long term debt in the following way. The “cost” of a given amount of long
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term debt $D_L$ is the difference between the expected period 2 earnings and the situation in which there is no long term debt issuance, i.e. $A(D_L) - A(0)$. The “benefit” of a given amount of long term debt is defined inversely. It is the difference between the expected rolling over cost from no long term debt issuance to the expected cost from long term debt issuance $D_L$, i.e. $B(0) - B(D_L)$. Figures 7 to 9 plots the functions:

$$Cost(D_L) = [100\pi_0 - D_0] - [100\pi_0 - D_0 - (\pi_0 - p_L(D_L))D_L]$$

$$Benefit(D_L) = \int_{\pi(0) \geq 0} [\pi(\pi, 0) - C(x(\pi, 0))]dF(\pi) - \int_{\pi(\pi, D_L) \geq 0} [\pi(\pi, D_L) - C(x(\pi, D_L))]dF(\pi)$$

Figure 7: Cost & benefit of long-term debt
Figure 7 is a 3-dimensional representation of Cost and Benefits of every long term debt level for any possible income. Figure 8 and 9 plot benefit and cost for three income levels, corresponding to a low, a middle and a high income country respectively. The Figures show the three scenarios described in the previous section.

**Scenario 1: High Income Countries**
Governments of high income countries will never face a rollover crisis because $\pi Y - D_0 > 0$. From a rollover benefit perspective, they are indifferent on choosing any amount of $D_L$.

This is clearly seen in figure 8, where the benefit graph is flat for high income. However, as it is shown in figure 9, risk averse bondholders demand a risk premium for holding long term debt. Therefore any issuance of long term debt is sub-optimal for the government because it would raise the costs of borrowing. In consequence, all high income governments choose a short term debt maturity structure.

**Scenario 2: Middle Income Countries**

These governments have rollover risk because $\pi Y - D_0 < 0$. From a “Benefit” perspective, they would maximize their utility by rolling over all of $D_0$ using long term debt. As both figure 8 and figure 9 show, for a range of income and a given income respectively, the benefit is monotonically increasing in long term debt. Still, by lemma 2, they cannot go too high in issuance because premiums go up. Thus, they face a trade-off between safer long term debt and cheaper short term debt. From section 4 we know it is optimal for a government facing this trade off, to issue a positive amount of long term debt.

**Scenario 3: Low Income Countries**
Like the previous case, governments in these economies also face rollover risk. They differ from middle income economies because they no longer minimize the expected rolling over cost by financing the totality of $D_0$ with long term debt. This is clearly shown in figure 8. Benefits from long term debt increase until certain level of $D_L$ to then decrease. The Benefit is even negative for high levels of long term debt.

As a result of faster increasing premiums, the decrease of period 2 earnings are higher low income economies. These economies end up with lower available resources to rollover debt in period one (because most income is used to repay long term debt). The probability of a rollover crisis thus increases. This, in addition to a more expensive long term debt makes governments of low income economies choose shorter maturities than governments of middle income ones.

6. Conclusions

This paper establishes that there is not a monotonic relationship between debt maturity structure and a country’s income. Low income economies rely more heavily on short term debt than middle income ones. Middle income economies, nonetheless, have a longer maturity structure than high income economies. Formally, it finds that a quadratic functional form fits the data for developing countries, while shares of short term debt are similar among developed nations.

It explains that the optimal debt maturity choice results from a trade-off between safer long term debt and cheaper short term debt. It shows that governments who choose their debt maturity structure according to this trade-off, prefer a short term maturity structure if they belong to the high income economy group, a long term maturity structure if they belong to the middle income economy group, and a short term maturity structure if they belong to the low income economy group. Moreover, the model predicts a similar debt maturity structure for high income economies.

This work provides an appropriate explanation and intuition behind all results. It established that the governments of high income countries, facing a low probability of rollover crisis in period 1, prefer the cheaper type of debt. Governments of middle income economies, who are more likely to face a rollover crisis, choose a longer maturity to ensure against this risk. While poor economies who are unstable prefer to avoid the cost of expensive long term debt and thus choose a short maturity structure.
References


A. Appendix

Volatility of income

In order to justify the assumption that governments of low income economies are more unstable in the sense that they have greater variance in expected income than the governments of high income economies, empirical evidence has been collected to show that this is in fact true.

Using data on income per capita for the 97 countries considered in section 2 of the paper from 1990 on I find that the pattern observed is the one exhibited in figure.

![Figure 13: Income volatility](image)

Proofs of lemmas

Proof of Lemma 1:

- **Lemma 1**: If long term debt is positive, $D_L > 0$, the price of long term is $p(D_L) < 0$. If $D_L = 0$, then the price of long term debt is $p(D_L) = \pi_0$.

First part:

WTS: If long term debt is positive, $D_L > 0$, the price of long term is $p(D_L) < 0$.

$E[u'(w + (\pi - p_L)D_L)(\pi_0 - p_L)] = 0$ by the linearity of the expected value operator,

$E[u'(w + (\pi - p_L)D_L)\pi_0] = p_LE[u'(w + (\pi_0 - p_L)D_L)]$ then,

$p_L = E[u'(w + (\pi_0 - p_L)D_L)\pi]/E[u'(w + (\pi - p_L)D_L)]$ then,

$p_L < \pi_0$ iff $E[u'(w + (\pi - p_L)D_L)\pi] < E[u'(w + (\pi - p_L)D_L)\pi_0]$

$E[u'(w + (\pi - p_L)D_L)\pi] = E[u'(w + (\pi - p_L)D_L)]E[\pi] + Cov(u'(w + (\pi - p_L)D_L), \pi)$ then,

$E[u'(w + (\pi - p_L)D_L)\pi_0] + Cov(u'(w + (\pi - p_L)D_L), \pi)$

$Cov(u'(w + (\pi - p_L)D_L), \pi) < 0$ because $u''(\cdot)$ is strictly decreasing and $w + (\pi - p_L)D_L$ is in-
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...creasing in $\pi$.

Then $E[u'(w + (\pi - pL)DL)\pi] < E[u'(w + (\pi - pL)DL)]\pi_0$ and thus $pL < \pi_0$.

Second part:

WTS: If $D_L = 0$, then the price of long term debt is $p(D_L) = \pi_0$.

When $D_L = 0$, $E[u'(w + (\pi - pL)DL)(\pi - pL)] = E[u'(w)(\pi - pL)] = 0$. Since $u'(w)$ is a constant, $p_L = \pi_0$.

**Proof of Lemma 2:**

I prove this theorem for utility functions with constant relative risk aversion lower or equal to one. Broner, Lorenzoni and Schmukler (WP 2006) demonstrate this theorem for utility functions with decreasing absolute risk aversion. Their demonstration can easily be extended to constant absolute risk aversion utility functions.

Proof for utility functions with constant relative risk aversion lower or equal to one:

WTS: $P_L D_L$ is decreasing in $D_L$.

$E[u'(w + (\pi - p_L^0)D_L^0)(\pi - p_L^0)] = 0$

$\int_0^{p_L^0} u'(w + (\pi - p_L^0)D_L^0)(\pi - p_L^0)dF(\pi) + \int_{p_L^0}^{1} u'(w + (\pi - p_L^0)D_L^0)(\pi - p_L^0)dF(\pi) = 0$

Suppose $D_L^0$ increases from $D_L^0$ to $D_L^1$.

In the first region, $\pi - p_L^0 < 0$. Then, $w + (\pi - p_L)D_L^0 > w + (\pi - p_L)D_L^1$.

Then $u'(w + (\pi - p_L)D_L^0) < u'(w + (\pi - p_L)D_L^1)$

Analogously, for second region $u'(w + (\pi - p_L)D_L^0) > u'(w + (\pi - p_L)D_L^1)$

Then, $\int_0^{p_L^0} u'(w + (\pi - p_L^0)D_L^0)(\pi - p_L^0)dF(\pi) + \int_{p_L^0}^{1} u'(w + (\pi - p_L^0)D_L^0)(\pi - p_L^0)dF(\pi) < 0$

Suppose $p_L$ goes from $p_L^0$ to $p_L^1$, where $p_L^1 > p_L^0$. In order to return to equilibrium (FOCs=0) the last integral must increase for this change in price.

The last integral can be written in the following way:

$\int_0^{p_L^0} u'(w + (\pi - p_L^0)D_L^0)(\pi - p_L^0)dF(\pi) + \int_{p_L^0}^{1} u'(w + (\pi - p_L^0)D_L^0)(\pi - p_L^0)dF(\pi) + \int_{p_L^0}^{1} u'(w + (\pi - p_L^0)D_L^1)(\pi - p_L^1)dF(\pi) < 0$

Analyzing the three integrals separately:

$\int_0^{p_L^0} u'(w + (\pi - p_L^0)D_L^0)(\pi - p_L^0)dF(\pi) > \int_0^{p_L^0} u'(w + (\pi - p_L^0)D_L^1)(\pi - p_L^1)dF(\pi)$ [because $\pi - p_L^0 > \pi - p_L^1$ and $u'(:) > 0$] $> \int_0^{p_L^0} u'(w + (\pi - p_L^1)D_L^1)(\pi - p_L^1)dF(\pi)$

[because $u'(:)$ is decreasing].
Analogously,

\[ \int_{p_2^L}^{p_2^L} u'(w + (\pi - p_2^L)D_2^L)(\pi - p_2^L)dF(\pi) > \int_{p_2^L}^{p_2^L} u'(w + (\pi - p_2^L)D_2^L)(\pi - p_2^L)dF(\pi). \]

The third integral

\[ \int_{p_2^L}^{p_2^L} u'(w + (\pi - p_2^L)D_2^L)(\pi - p_2^L)dF(\pi) > \int_{p_2^L}^{p_2^L} u'(w + (\pi - p_2^L)D_2^L)(\pi - p_2^L)dF(\pi) \iff -u''(w + (\pi - p_2^L)D_2^L)(\pi - p_2^L)D_2^L \leq 0 \iff \frac{u''(w)}{u'(w)}[w + (\pi - p_2^L)D_2^L] \leq 1 \]

and this is true by assumption.

Then, an increase in the price makes the first order conditions even smaller. Then the price must decrease.